ALPHALEARN STUDYLAB

STD 8

MATHEMATICS

Ganita Prakash — Publication-Grade Practice Worksheets

Part I & Part II

Based on CBSE / NCERT Curriculum | Answer Keys Included

STD 8 — MATHEMATICS

Publication-Grade Practice Worksheets | AlphaLearn Studylab

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Ganita Prakash

PART I

CHAPTER 1

A Square and a Cube

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Std 8 Mathematics — Part 1

Chapter 1: A Square and a Cube

Worksheet 1 — Concept Builder

Student Name: ______________________________	Class: __________ Section: __________
Roll No.: __________	Date: __________________

Maximum Marks: 50 **Time Allowed: 75 minutes**

General Instructions:

1. This worksheet contains 39 questions divided into Sections A to I.
2. All questions are compulsory unless stated otherwise.
3. Show complete working for all numerical questions.
4. Use of calculators is not permitted.
5. Write your final answers clearly and neatly in the space provided.

Section A — Multiple Choice Questions (1 mark each)

1. Which of the following is a perfect square? *[1]*
(a) 45 (b) 64 (c) 90 (d) 125

2. The units digit of a perfect square can never be: *[1]*
(a) 6 (b) 4 (c) 8 (d) 9

3. The number of zeros at the end of 300^2 is: *[1]*
(a) 2 (b) 3 (c) 4 (d) 6

4. Which of the following is a perfect cube? *[1]*
(a) 100 (b) 125 (c) 150 (d) 200

5. $15^2 - 14^2$ is equal to: *[1]*
(a) 27 (b) 29 (c) 31 (d) 33

6. The cube root of 1728 is: *[1]*
(a) 10 (b) 11 (c) 12 (d) 13

7. Which of these numbers is definitely NOT a perfect square (using the units-digit test)? *[1]*
(a) 361 (b) 484 (c) 733 (d) 900

8. The square root of a perfect square that ends in 4 must end in: *[1]*
(a) 2 or 8 (b) 4 or 6 (c) 3 or 7 (d) 1 or 9

9. Which of the following numbers is both a perfect square and a perfect cube? *[1]*
(a) 64 (b) 100 (c) 125 (d) 144

Section B — Fill in the Blanks (1 mark each)

1. The square of an even number is always __________. *[1]*

2. 7^2 = __________. *[1]*

3. The sum of the first 6 odd numbers is __________. *[1]*

4. $\sqrt{225}$ = __________. *[1]*

5. A number is a perfect cube if its prime factors can be grouped into __________ identical groups. *[1]*

6. $(-6)^3 =$ __________. *[1]*

Section C — True or False (Correct the false statements) (1 mark each)

1. Every perfect square ends in 1, 4, 5, 6, 9, or an even number of zeros. *[1]*

2. The square root of every perfect square has only one possible value. *[1]*

3. 128 is a perfect cube. *[1]*

4. The cube of a negative number is always negative. *[1]*

5. 800 is a perfect cube. *[1]*

Section D — Match the Following (5 marks)

Match each entry in Column A with its correct value in Column B. Write your answers as (a)–(__), (b)–(__), etc.

Column A	Column B
(a) 6^2	(i) 25
(b) $\sqrt[3]{27}$	(ii) 14
(c) 12^2	(iii) 3
(d) Sum of first 5 odd numbers	(iv) 144
(e) $\sqrt{196}$	(v) 36

Section E — Very Short Answer Questions (1 mark each)

1. Write the square of 25. *[1]*

2. Is 361 a perfect square? Answer using the units-digit test, and state whether this test alone is conclusive. *[1]*

3. Find $\sqrt[3]{64}$. *[1]*

4. Write the number of zeros in the square of 5000. *[1]*

5. Express 49 as a sum of consecutive odd numbers starting from 1. *[1]*

Section F — Basic Numerical Problems (2 marks each)

1. Find the square root of 1024 using prime factorisation. *[2]*

2. Check whether 3136 is a perfect square using prime factorisation. *[2]*

3. Find the cube root of 2744 using prime factorisation. *[2]*

4. Using the method of successive subtraction of consecutive odd numbers, verify whether 121 is a perfect square. *[2]*

Section G — Identify the Mistake (2 marks each)

1. A student wrote: "$\sqrt{81} = 9$ only, because the square root of a positive number always has one value." Identify the mistake in this statement and write the correct answer. *[2]*

2. A student calculated 45^2 as: $45^2 = (40 + 5)^2 = 40^2 + 5^2 = 1600 + 25 = 1625$. Identify the mistake in this working and find the correct value of 45^2. *[2]*

Section H — Complete the Pattern (2 marks each)

1. Study the pattern: $1^2 + 2^2 + 2^2 = 3^2$, $2^2 + 3^2 + 6^2 = 7^2$, $3^2 + 4^2 + 12^2 = 13^2$. Complete the next line: $4^2 + 5^2 + 20^2 = (\_\_\_\_)^2$ *[2]*

2. Complete: $1 + 3 + 5 + 7 + 9 + 11 + 13 = (\_\_\_\_)^2$ *[2]*

Section I — Concept Check (Reasoning) (2 marks each)

1. Without performing full multiplication, explain why 2453 cannot be a perfect square. *[2]*

2. Explain, with the idea of 'partner factors', why every perfect square has an odd number of factors. *[2]*

ANSWER KEY — Worksheet 1: Concept Builder

Section A — MCQs

1. (b) 64

2. (c) 8

3. (c) 4 [$300^2 = 90000$]

4. (b) 125

5. (b) 29 [$15^2-14^2=(15-14)(15+14)=29$]

6. (c) 12

7. (c) 733

8. (b) 4 or 6

9. (a) 64 [$64 = 8^2 = 4^3$]

Section B — Fill in the Blanks

1. even

2. 49

3. 36 [1+3+5+7+9+11]

4. 15

5. three

6. −216

Section C — True / False

1. True.

2. False. Every perfect square has TWO integer square roots: one positive and one negative (e.g. $\sqrt{64} = \pm 8$).

3. False. $128 = 2^7$; its prime factors cannot be split into three identical groups, so 128 is not a perfect cube. (The nearest perfect cubes are $125 = 5^3$ and $216 = 6^3$.)

4. True. A negative number multiplied by itself three times remains negative, e.g. $(-5)^3 = -125$.

5. False. $800 = 2^5 \times 5^2$; the factors cannot be split into three identical groups, so 800 is not a perfect cube.

Section D — Match the Following

(a)–(v) 36 (b)–(iii) 3 (c)–(iv) 144 (d)–(i) 25 (e)–(ii) 14

Section E — Very Short Answer

1. $25^2 = 625$

2. 361 ends in 1, which is a possible units digit for a perfect square, but this test alone is not conclusive (e.g. 371 also ends in 1 but is not a square). Checking further: $19^2 = 361$, so 361 IS a perfect square.

3. $\sqrt[3]{64} = 4$ [since $4^3 = 64$]

4. 6 zeros [$5000^2 = 25,000,000$]

5. 49 = 1 + 3 + 5 + 7 + 9 + 11 + 13

Section F — Basic Numerical Problems

1. $1024 = 2^{10} = (2^5)^2 = 32^2$. So $\sqrt{1024} = 32$.

2. $3136 = 2^6 \times 7^2 = (2^3 \times 7)^2 = 56^2$. Since the prime factors split into two identical groups ($2^3 \times 7$ each), 3136 is a perfect square, and $\sqrt{3136} = 56$.

3. $2744 = 2^3 \times 7^3 = (2 \times 7)^3 = 14^3$. So $\sqrt[3]{2744} = 14$.

4. 121 − 1 = 120, 120 − 3 = 117, 117 − 5 = 112, 112 − 7 = 105, 105 − 9 = 96, 96 − 11 = 85, 85 − 13 = 72, 72 − 15 = 57, 57 − 17 = 40, 40 − 19 = 21, 21 − 21 = 0. Since 0 is reached exactly after subtracting 11 consecutive odd numbers, $121 = 11^2$ is a perfect square.

Section G — Identify the Mistake

1. Mistake: A perfect square actually has TWO integer square roots, +9 and −9, since $9^2 = 81$ and $(-9)^2 = 81$ as well. Correct answer: $\sqrt{81} = \pm 9$ (by the convention used in this chapter, the positive root 9 is taken).

2. Mistake: The student forgot the middle term of the identity $(a+b)^2 = a^2 + 2ab + b^2$. Correct working: $45^2 = (40+5)^2 = 40^2 + 2\times40\times5 + 5^2 = 1600 + 400 + 25 = 2025$.

Section H — Complete the Pattern

1. $4^2 + 5^2 + 20^2 = 16 + 25 + 400 = 441 = 21^2$. So the missing number is 21.

2. $1+3+5+7+9+11+13 = 49 = 7^2$ (sum of the first 7 odd numbers equals 7^2).

Section I — Concept Check (Reasoning)

1. 2453 ends in the digit 3. Perfect squares can only end in 0, 1, 4, 5, 6, or 9. Since 3 is not among these digits, 2453 cannot be a perfect square.

2. Every factor of a number pairs with a 'partner factor' whose product gives the number. For most numbers these pairs are distinct, giving an even count of factors. For a perfect square, one factor (its square root) is paired with itself, so it is counted only once instead of twice — making the total number of factors odd.

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Std 8 Mathematics — Part 1

Chapter 1: A Square and a Cube

Worksheet 2 — Skill Mastery

Student Name: ________________________________	Class: __________ Section: __________
Roll No.: __________	Date: __________________

Maximum Marks: 69 **Time Allowed: 90 minutes**

General Instructions:

1. This worksheet contains 26 questions across Sections A to G.
2. All questions require complete step-by-step working; final answers alone will not earn full marks.
3. Diagrams, where relevant, should be neat and labelled.
4. Attempt all questions in the order given.
5. Use of calculators is not permitted.

Section A — Numerical Problems (2 marks each)

1. Find the smallest number by which 1800 must be multiplied so that the product is a perfect square. Also find the square root of the resulting number. *[2]*

2. Find the smallest number by which 2560 must be divided so that the quotient is a perfect cube. *[2]*

3. Find the square root of 7744 using prime factorisation. *[2]*

4. Find the cube root of 15625 using prime factorisation. *[2]*

5. Evaluate $\sqrt{0.0049}$. *[2]*

6. Find the value of $(4/9)^2$ and $(4/9)^3$. *[2]*

Section B — Multi-Step Problems (3 marks each)

1. Given that $45^2 = 2025$, find the value of 46^2 without direct multiplication. *[3]*

2. A square garden has an area of 2704 m^2. Find the length of its side, and the cost of fencing it at ₹150 per metre. *[3]*

3. Find the smallest square number that is divisible by each of 6, 9 and 15. *[3]*

4. A cube-shaped water tank has a volume of 3375 m^3. Find the length of its edge, and state how many cube-shaped containers of volume 1 m^3 are needed to fill it. *[3]*

5. Using the fact that every square is a sum of consecutive odd numbers, find 15^2, given that $14^2 = 196$. *[3]*

Section C — Word Problems (3 marks each)

1. Rohan wants to arrange 1764 chairs into rows and columns such that the number of rows equals the number of columns. Find the number of chairs in each row, verifying your answer using prime factorisation. *[3]*

2. A housing society is planning a square-shaped park with an area of 5000 m^2. Since 5000 is not a perfect square, find the side length (as a whole number of metres) of the largest square park that can actually be built within this area. *[3]*

3. The Hardy–Ramanujan number 1729 can be written as the sum of two cubes in two different ways. One way uses 1^3 and 12^3. Show, with full working, that the other way uses 9^3 and 10^3. *[3]*

4. Using the pattern relating consecutive odd numbers to perfect cubes, find the value of 91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109 without adding the terms one by one. *[3]*

Section D — Logical Reasoning & HOTS (3 marks each)

1. Without fully factorising, explain why 2028 cannot be a perfect square. (Hint: use the units-digit test and one further check.) *[3]*

2. If a perfect square n^2 has exactly 5 digits, find the possible range of values of n. Justify your reasoning. *[3]*

3. A number ends with exactly four zeros. Can it be a perfect cube? Justify your answer with reasoning about zeros in cubes. *[3]*

4. Prove algebraically that the difference between the squares of any two consecutive natural numbers, n and (n+1), is always an odd number. Verify your result using n = 3. *[3]*

Section E — Pattern Recognition (2 marks each)

1. Continue the pattern: $1^2+2^2+2^2=3^2$, $2^2+3^2+6^2=7^2$, $3^2+4^2+12^2=13^2$, $4^2+5^2+20^2=(\_\_)^2$. Find the missing number. *[2]*

2. Continue the same pattern one more step to find the missing numbers: $9^2 + 10^2 + (\_\_\_\_)^2 = (\_\_\_\_)^2$ *[2]*

3. Compute successive differences of the perfect cubes 1, 8, 27, 64, 125, 216 level by level until the differences become constant. Show all levels. *[2]*

Section F — Case-Based Questions (4 marks each)

1. Case Study: In a locker room with lockers numbered 1 to 50, Person 1 opens every locker, Person 2 toggles every 2nd locker, Person 3 toggles every 3rd locker, and so on up to Person 50, exactly as in the classic locker puzzle. *[4]*

(i) How many times is locker number 24 toggled? List its factors.
(ii) Will locker number 36 remain open or closed at the end? Justify using factor pairs.
(iii) List all locker numbers from 1 to 50 that remain open at the end.

2. Case Study: A tailor has a square piece of cloth of area 200 cm². She wants to cut out the largest possible square handkerchief with an integer side length. *[4]*

(i) Is 200 a perfect square? Show the two nearest perfect squares to 200.
(ii) Find the side length of the largest handkerchief (integer cm) she can cut.
(iii) Find the area of cloth left over (wasted) after cutting this handkerchief.

Section G — Real-Life Application (2 marks each)

1. An architect is designing a cube-shaped decorative box of volume 4096 cm³. Find the length of each edge of the box. *[2]*

2. A farmer has a rectangular field of area 1156 m² that he wants to convert into a square field of the same area. Find the side length of the square field. *[2]*

ANSWER KEY — Worksheet 2: Skill Mastery

Section A — Numerical Problems

1. $1800 = 2^3\times3^2\times5^2$. The prime 2 appears 3 times (unpaired one 2), so multiply by 2 to make it $2^4\times3^2\times5^2 = (2^2\times3\times5)^2 = 60^2$. Smallest multiplier = 2; resulting number = 3600; $\sqrt{3600} = 60$.

2. $2560 = 2^9\times5$. To make exponents multiples of 3: 2^9 is already fine ($9=3\times3$), but 5^1 is not a multiple of 3, so divide by 5. Quotient = $512 = 2^9 = 8^3$. Smallest divisor = 5.

3. $7744 = 2^6\times1^{12}$... precisely $7744 = 2^6 \times 11^2 = (2^3\times11)^2 = 88^2$. So $\sqrt{7744} = 88$.

4. $15625 = 5^6 = (5^2)^3$ehm: $5^6 = (5^2)^3 = 25^3$. So $\sqrt[3]{15625} = 25$.

5. $0.0049 = 49/10000$. $\sqrt{49/10000} = 7/100 = 0.07$.

6. $(4/9)^2 = 16/81$. $(4/9)^3 = 64/729$.

Section B — Multi-Step Problems

1. $46^2 = 45^2$ + (45th odd number's successor) $= 45^2 + 2\times45 + 1 = 2025 + 90 + 1 = 2116$.

2. $2704 = 2^4\times13^2 = 52^2$, so side = 52 m. Perimeter = 4×52 = 208 m. Cost = $208 \times$ ₹150 = ₹31,200.

3. LCM of 6, 9, 15 = $90 = 2\times3^2\times5$. To make it a perfect square, each prime needs an even power: 2 needs one more 2, 5 needs one more 5. Multiply by $2\times5=10$: $90\times10 = 900 = 30^2$. Smallest square number = 900.

4. $3375 = 3^3\times5^3 = 15^3$, so edge = 15 m. Volume = 3375 m³, so 3375 containers of 1 m³ each are needed.

5. $15^2 = 14^2$ + 29th odd number $= 196 + (2\times15-1) = 196 + 29 = 225$.

Section C — Word Problems

1. $1764 = 2^2\times3^2\times7^2 = (2\times3\times7)^2 = 42^2$. So each row (and column) has 42 chairs.

2. The nearest perfect squares to 5000 are $70^2 = 4900$ and $71^2 = 5041$. Since $4900 < 5000$, the largest square park with an integer side length that fits is 70 m × 70 m.

3. $9^3 = 729$ and $10^3 = 1000$. Sum = 729 + 1000 = 1729, confirming $1729 = 9^3 + 10^3$ (matching the second representation, alongside $1^3 + 12^3 = 1 + 1728 = 1729$).

4. This is a run of 10 consecutive odd numbers starting from 91 = the 46th odd number (since $2\times46-1=91$) and ending at 109 = the 55th odd number. The sum of the first n odd numbers is n^2, so this partial sum = (sum of first 55 odd numbers) − (sum of first 45 odd numbers) $= 55^2 - 45^2 = 3025 - 2025 = 1000$.

Section D — Logical Reasoning & HOTS

1. 2028 ends in 8, and no perfect square ends in 8 — this alone proves 2028 cannot be a perfect square.

2. A 5-digit number lies between 10000 and 99999. Since $99^2=9801$ (4 digits) and $100^2=10000$ (5 digits), and $316^2=99856$, $317^2=100489$, n must satisfy $100 \le n \le 316$ for n^2 to have exactly 5 digits.

3. No. A perfect cube can only end in a number of zeros that is a multiple of 3 (since each zero in the cube root contributes 3 zeros to the cube). Four zeros is not a multiple of 3, so such a number cannot be a perfect cube.

4. $(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$. Since 2n is always even, 2n+1 is always odd — so the difference is always odd, for every natural number n. Verification for n=3: $4^2 - 3^2 = 16 - 9 = 7$, which is odd and equals 2(3)+1 = 7. ✓

Section E — Pattern Recognition

1. $4^2+5^2+20^2 = 16+25+400 = 441 = 21^2$. Missing number = 21.

2. $9^2+10^2+90^2 = 81+100+8100 = 8281 = 91^2$. So the missing numbers are 90 and 91.

3. Level 1 (first differences): 7, 19, 37, 61, 91. Level 2 (second differences): 12, 18, 24, 30. Level 3 (third differences): 6, 6, 6 — constant. The differences become constant at the third level for perfect cubes.

Section F — Case-Based Questions

1. (i) Locker 24 is toggled 8 times (factors of 24: 1, 2, 3, 4, 6, 8, 12, 24). (ii) Locker 36 remains OPEN: $36 = 6^2$ is a perfect square, so it has an odd number of factors (9 factors: 1,2,3,4,6,9,12,18,36), leaving it toggled an odd number of times. (iii) Open lockers from 1–50: 1, 4, 9, 16, 25, 36, 49.

2. (i) 200 is not a perfect square; the nearest perfect squares are $14^2=196$ and $15^2=225$. (ii) Largest integer side length = 14 cm. (iii) Area used = 14^2 = 196 cm²; wasted area = 200 − 196 = 4 cm².

Section G — Real-Life Application

1. $4096 = 2^{12} = (2^4)^3 = 16^3$. Edge length = 16 cm.

2. $1156 = 2^2 \times 17^2 = 34^2$. Side length of the square field = 34 m.

AlphaLearn Studylab

Std 8 Mathematics — Part 1

Chapter 1: A Square and a Cube

Worksheet 3 — Exam Mastery

Student Name: ______________________________	Class: __________ Section: __________
Roll No.: __________	Date: __________________

Maximum Marks: 50 **Time Allowed: 90 minutes**

General Instructions:

1. This question paper contains six sections: A, B, C, D, E and F.
2. Section A has 8 MCQs of 1 mark each. Section B has 2 Assertion-Reason questions of 1 mark each.
3. Section C has 5 Very Short Answer questions of 2 marks each.
4. Section D has 4 Short Answer questions of 3 marks each. Section E has 2 Long Answer questions of 5 marks each.
5. Section F has 2 Case-Based questions of 4 marks each.
6. All questions are compulsory. Show complete working for full marks.

Section A — Multiple Choice Questions (1 × 8 = 8 marks)

1. Which of the following numbers is a perfect square? *[1]*
(a) 800 (b) 900 (c) 700 (d) 600

2. $\sqrt{576}$ = ? *[1]*
(a) 22 (b) 24 (c) 26 (d) 28

3. The cube root of −343 is: *[1]*
(a) −7 (b) 7 (c) −49 (d) 49

4. The number of zeros at the end of 70^2 is: *[1]*
(a) 1 (b) 2 (c) 3 (d) 4

5. Which of these numbers definitely cannot be a perfect square, based on its units digit? *[1]*
(a) 1444 (b) 1447 (c) 1521 (d) 1600

6. If $x^3 = 1000$, then x = ? *[1]*
(a) 10 (b) 100 (c) 20 (d) 30

7. $99^2 = 100^2 - ?$ *[1]*
(a) 99 (b) 199 (c) 198 (d) 200

8. Which of these is a number that can be written as the sum of two cubes in two different ways, as discussed in the chapter? *[1]*
(a) 1000 (b) 1729 (c) 1331 (d) 512

Section B — Assertion & Reason (1 × 2 = 2 marks)

For questions 1 and 2, two statements are given — Assertion (A) and Reason (R). Choose the correct option:

(a) Both A and R are true, and R is the correct explanation of A. (b) Both A and R are true, but R is NOT the correct explanation of A. (c) A is true, but R is false. (d) A is false, but R is true.

1. Assertion (A): 196 is a perfect square. Reason (R): The prime factorisation of 196 is $2^2 \times 7^2$, which can be split into two identical groups. *[1]*

2. Assertion (A): The cube of every odd number is odd. Reason (R): The product of odd numbers is always odd. *[1]*

Section C — Very Short Answer Questions (2 × 5 = 10 marks)

1. Find the square root of 289 using prime factorisation. *[2]*

2. Write the next number in the pattern of perfect cubes: 1, 8, 27, 64, ____. *[2]*

3. Is 1000 a perfect cube? Justify your answer with working. *[2]*

4. Find 51^2 using the identity $(50 + 1)^2$. *[2]*

5. Write any two perfect square numbers that lie between 200 and 300. *[2]*

Section D — Short Answer Questions (3 × 4 = 12 marks)

1. Using prime factorisation, determine whether 3969 is a perfect square. If it is, find its square root. *[3]*

2. Find the cube root of 9261 using prime factorisation. *[3]*

3. Using the property of successive subtraction of consecutive odd numbers, verify whether 100 is a perfect square. Show all steps. *[3]*

4. The volume of a cube is 2197 cm^3. Find the length of its edge, and hence find its total surface area ($6 \times side^2$). *[3]*

Section E — Long Answer Questions (5 × 2 = 10 marks)

1. A school wants to construct a square-shaped assembly ground of area 3600 m^2. *[5]*

(a) Find the side length of the ground.
(b) The school also wants to build a cubical storage tank whose volume (in m^3) equals the cube of one-fifth of the ground's side length (in metres). Find the edge length of this tank.
(c) Find the total cost of fencing the assembly ground at ₹80 per metre.

2. Using prime factorisation, determine whether 11025 is a perfect square. If it is, find its square root. Then verify your answer using the estimation method (narrowing the interval using nearby known squares), explaining each step clearly, as shown in the chapter. *[5]*

Section F — Case-Based Questions (4 × 2 = 8 marks)

Case Study 1 — The Locker Puzzle

In a room with 100 lockers numbered 1 to 100, Person 1 opens every locker, Person 2 toggles every 2nd locker, Person 3 toggles every 3rd locker, and so on, with Person k toggling every k-th locker, until all 100 people have taken their turn. A locker is toggled once for every factor of its locker number.

(i). How many times is locker number 12 toggled? List its factors. *[1]*

(ii). Will locker number 49 remain open or closed at the end? Justify your answer. *[1]*

(iii). List all the locker numbers between 1 and 30 that remain open at the end. *[1]*

(iv). What general mathematical property explains why only square-numbered lockers remain open? *[1]*

Case Study 2 — The Hardy–Ramanujan Number

The mathematician Srinivasa Ramanujan famously identified 1729 as the smallest number that can be expressed as the sum of two cubes in two different ways. Such numbers are called taxicab numbers.

(i). Express 1729 as a sum of two cubes in each of its two ways, showing the working. *[1]*

(ii). In your own words, what is a ‘taxicab number’? *[1]*

(iii). Verify numerically that $9^3 + 10^3 = 1729$. *[1]*

(iv). Name the mathematician who was travelling in the taxicab and had the conversation with Ramanujan about the number 1729. *[1]*

ANSWER KEY — Worksheet 3: Exam Mastery

Section A — MCQs

1. (b) 900

2. (b) 24

3. (a) −7

4. (b) 2 [$70^2=4900$]

5. (b) 1447 [ends in 7]

6. (a) 10

7. (b) 199 [$100^2-99^2=(100-99)(100+99)=199$]

8. (b) 1729

Section B — Assertion & Reason

1. (a) Both A and R are true, and R is the correct explanation of A.

2. (a) Both A and R are true, and R is the correct explanation of A — since an odd number's prime factorisation contains no factor of 2, its cube (a product of three odd numbers) also has no factor of 2 and so remains odd.

Section C — Very Short Answer

1. $289 = 17^2$, so $\sqrt{289} = 17$.

2. 125 [since $5^3 = 125$, continuing $1^3, 2^3, 3^3, 4^3, 5^3$]

3. Yes. $1000 = 10^3 = (2\times5)^3 = 2^3\times5^3$, and since the prime factors split into three identical groups, 1000 is a perfect cube; $\sqrt[3]{1000} = 10$.

4. $51^2 = (50+1)^2 = 50^2 + 2\times50\times1 + 1^2 = 2500 + 100 + 1 = 2601$.

5. Any two of: 225 (15^2), 256 (16^2), 289 (17^2).

Section D — Short Answer

1. $3969 = 3^4 \times 7^2 = (3^2\times7)^2 = 63^2$. Since the prime factors split into two identical groups, 3969 IS a perfect square, and $\sqrt{3969} = 63$.

2. $9261 = 3^3 \times 7^3 = (3\times7)^3 = 21^3$. So $\sqrt[3]{9261} = 21$.

3. 100−1=99, 99−3=96, 96−5=91, 91−7=84, 84−9=75, 75−11=64, 64−13=51, 51−15=36, 36−17=19, 19−19=0. Zero is reached exactly after subtracting the first 10 odd numbers, so $100 = 10^2$ is a perfect square.

4. $2197 = 13^3$, so edge length = 13 cm. Total surface area = $6 \times 13^2 = 6 \times 169 = 1014$ cm².

Section E — Long Answer

1. (a) $3600 = 60^2$, so side = 60 m. (b) One-fifth of the side = $60\div5 = 12$ m; volume of tank = $12^3 = 1728$ m³, so the edge length of the tank = $\sqrt[3]{1728} = 12$ m. (c) Perimeter = $4\times60 = 240$ m; Cost = 240 × ₹80 = ₹19,200.

2. $11025 = 3^2 \times 5^2 \times 7^2 = (3\times5\times7)^2 = 105^2$. So 11025 is a perfect square and $\sqrt{11025} = 105$. Estimation check: 11025 lies between $100^2=10000$ and $110^2=12100$, so $100 < \sqrt{11025} < 110$. Since 11025 ends in 5, its square root must end in 5, giving the candidate 105. Checking: $105^2 = (100+5)^2 = 10000+1000+25 = 11025$ ✓, confirming $\sqrt{11025} = 105$.

Section F — Case-Based Questions

1(i). Locker 12 is toggled 6 times. Factors of 12: 1, 2, 3, 4, 6, 12.

1(ii). Locker 49 remains OPEN, because $49 = 7^2$ is a perfect square and therefore has an odd number of factors (1, 7, 49 — 3 factors), so it is toggled an odd number of times.

1(iii). Open lockers from 1–30: 1, 4, 9, 16, 25 (all perfect squares in this range).

1(iv). Every factor of a number pairs with a partner factor; these pairs are distinct except when a number is a perfect square, where one factor pairs with itself. This makes the total factor count odd only for perfect squares, so only square-numbered lockers are toggled an odd number of times and remain open.

2(i). $1729 = 1^3 + 12^3 = 1 + 1728 = 1729$, and also $1729 = 9^3 + 10^3 = 729 + 1000 = 1729$.

2(ii). A taxicab number is a number that can be expressed as the sum of two positive cubes in two (or more) genuinely different ways.

2(iii). $9^3 + 10^3 = 729 + 1000 = 1729$ ✔️

2(iv). G. H. Hardy.