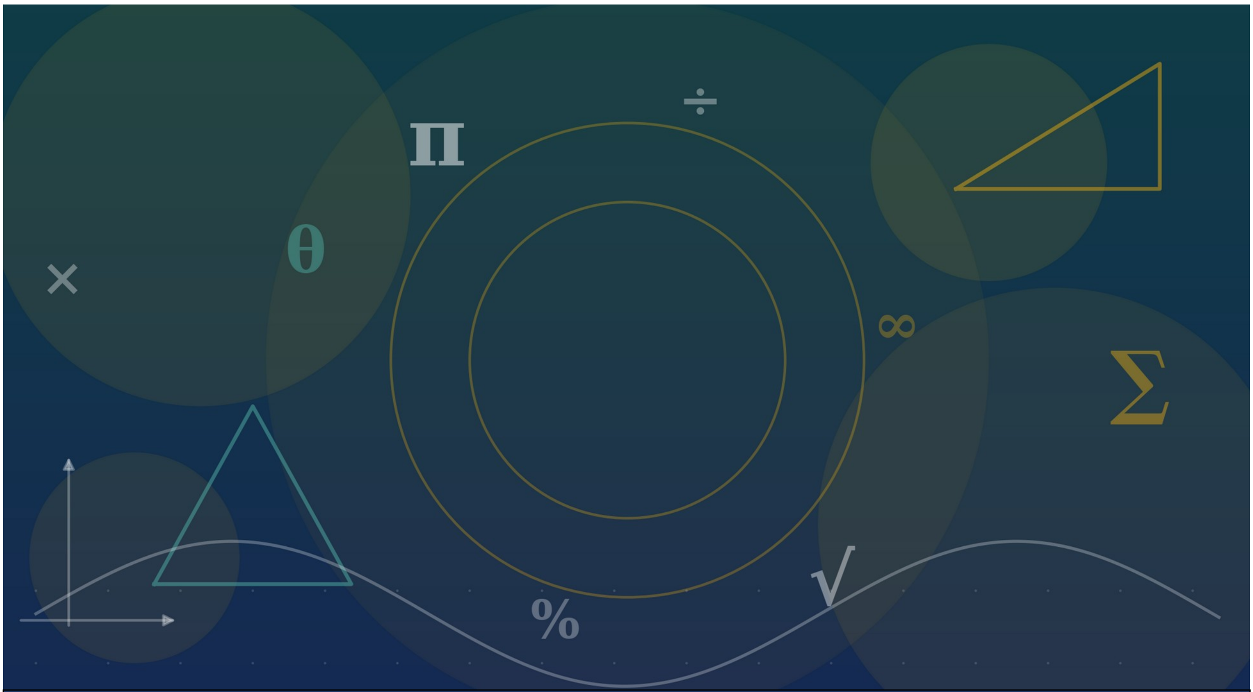

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Table of Contents

Grouped by CBSE Unit, exactly as the board weights them.

How to Use This Book

1	**Start with the Exam Radar** Every chapter opens with trend analysis and a 2027 prediction — read this FIRST so you know exactly what the board rewards before you practise a single question.
2	**Learn the Concept Capsule cold** This is the entire chapter's theory compressed to essentials, with diagrams wherever geometry, trigonometry, or mensuration needs them. Revisit this the night before your exam.
3	**Work through every section in order** MCQ → Assertion-Reason → 2-mark → 3-mark → 5-mark → Case Studies → HOTS. The difficulty climbs deliberately — don't skip to HOTS before the basics are automatic.
4	**Cover the answer, solve, then check** Every answer is worked in full steps — no compressed one-line jumps. Attempt the question yourself first; use the steps to find exactly where your own method diverges.
5	**Track your progress** Use the Study Tracker on the next page to check off each chapter as you complete it. At one chapter a week, the full book is done in 14 weeks — with time left to revise before boards.

Your Chapter-wise Study Tracker

Tick each box as you finish it. A chapter counts as 'done' only once all four are checked.

Ch.	Chapter	Concepts Read	MCQ + AR Done	B/C/D Solved	HOTS Attempted
1	Real Numbers	☐	☐	☐	☐
2	Polynomials	☐	☐	☐	☐
3	Pair of Linear Equations	☐	☐	☐	☐
4	Quadratic Equations	☐	☐	☐	☐
5	Arithmetic Progressions	☐	☐	☐	☐
6	Triangles	☐	☐	☐	☐
7	Coordinate Geometry	☐	☐	☐	☐
8	Introduction to Trigonometry	☐	☐	☐	☐
9	Applications of Trigonometry	☐	☐	☐	☐
10	Circles	☐	☐	☐	☐
11	Areas Related to Circles	☐	☐	☐	☐
12	Surface Areas and Volumes	☐	☐	☐	☐
13	Statistics	☐	☐	☐	☐
14	Probability	☐	☐	☐	☐

Suggested 14-Week Countdown

One chapter a week finishes the entire syllabus in 14 weeks flat. Start with Chapter 1 the day you begin revision, and by Week 14 every chapter is behind you — leaving the final stretch before boards purely for full-length practice papers.

UNIT I — NUMBER SYSTEMS

≈6 marks • Chapter 1

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BOARD EXAM 2027

MOST IMPORTANT QUESTIONS BOOKLET

Chapter 1 : Real Numbers

44 Exam-Focused Questions • Full Step-wise Answer Key

Trend Analysis • MCQ • Assertion-Reason • Case Studies • HOTS

PART 1 ● EXAM RADAR — How the Board Tests This Chapter

1.1 Where This Chapter Sits in the 2027 Paper

Real Numbers is the only chapter of Unit I (Number Systems), which carries about 6 marks out of 80 in the theory paper. Small unit, but the marks are almost guaranteed — the board rarely skips this chapter, and the questions repeat familiar patterns year after year. This makes Real Numbers one of the highest return-on-effort chapters in the whole syllabus.

⚠️ Syllabus Alert (Very Important)

The current NCERT (rationalised edition) covers ONLY: the Fundamental Theorem of Arithmetic, HCF & LCM by prime factorisation, and proofs of irrationality.

DELETED from the syllabus: Euclid's Division Lemma/Algorithm and decimal expansions of rational numbers (terminating / non-terminating).

If you practise old question papers, SKIP questions on these deleted topics. Many old '1-mark decimal expansion' questions no longer appear.

1.2 Recent Board Trend (Indicative Pattern, 2023–2026)

Based on recent board papers and official sample papers, this is how Real Numbers has typically appeared:

Year	Typical Question Types	Topics Tested	Marks Seen
2023	MCQ + Short answer	HCF/LCM by prime factorisation; irrationality proof	1M + 2M/3M
2024	MCQ + Assertion-Reason + Proof	Prove √p type irrational; HCF×LCM relation	1M + 1M + 3M
2025	MCQ + Case study sub-part	LCM word problem (bells/tracks); composite number logic	1M + case-based
2026	MCQ + AR + Proof	Exponent-form HCF/LCM ($a = x^3y^2$ type); irrationality of $a + b\sqrt{p}$	1M + 1M + 2M/3M

Note: the table shows the recurring pattern, not a claim about every question in every set. Different sets vary, but these formats dominate.

1.3 Board's Favourite Topics (Ranked)

★★★ Rank 1 — Irrationality proofs. Prove √2, √3, √5 irrational; then prove 3 + 2√5, 7√5, 1/√2, 6 + √2 type numbers irrational. Appears almost every year as a 2, 3 or 5-mark question.

★★★ Rank 2 — HCF & LCM applications. Bells tolling together, runners on a circular track, stacking books, largest square tile, marching steps. Now the board's favourite raw material for Section E case studies.

★★ Rank 3 — Prime-factorisation logic. Can 6^n end with 0? Why is 7 × 11 × 13 + 13 composite? HCF/LCM when numbers are given in exponent form ($a = x^3y^2$, $b = xy^3$). Very common as MCQ and Assertion-Reason.

★ Rank 4 — HCF × LCM = product relation. Given HCF and one number (or product), find LCM — and the trap that this relation fails for three numbers.

1.4 Prediction for Board 2027

What to expect

• 1–2 questions in Section A: an MCQ on HCF/LCM or prime factorisation, plus a possible Assertion-Reason on irrationality or the HCF×LCM relation.

• 1 proof question in Section B/C (2 or 3 marks): 'Prove that $\sqrt{5}$ is irrational' or 'Prove that $3 + 2\sqrt{5}$ is irrational' style.

• Strong chance of an HCF/LCM real-life passage in Section E (case study).

Preparation rule: master ONE irrationality proof perfectly (the method is identical for all), and practise every word-problem type in this booklet.

PART 2 • CONCEPT CAPSULE — Full Theory in Simple Words

2.1 The Fundamental Theorem of Arithmetic (FTA)

Every composite number can be broken into a product of prime numbers, and this breakdown is unique — the only freedom you have is the order of writing the primes. For example, $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$, and there is no other set of primes that multiplies to 32760. Think of primes as the 'atoms' of numbers: every number is built from them in exactly one way.

2.2 HCF and LCM by Prime Factorisation

Step 1: Write each number as a product of prime powers.

HCF = product of the SMALLEST power of each COMMON prime.

LCM = product of the GREATEST power of EVERY prime that appears in any of the numbers.

Example: $6 = 2 \times 3$ and $20 = 2^2 \times 5$. HCF = 2 (only common prime, smallest power). LCM = $2^2 \times 3 \times 5 = 60$.

2.3 The Golden Relation

Memorise

For any TWO positive integers a and b: HCF(a, b) × LCM(a, b) = a × b

This is NOT true for three numbers. Example: HCF(6, 72, 120) = 6 and LCM(6, 72, 120) = 360, but 6 × 360 ≠ 6 × 72 × 120.

2.4 The Key Theorem Behind Irrationality Proofs

If p is a prime number and p divides a^2, then p must also divide a (where a is a positive integer). Reason: the primes of a^2 are just the primes of a, each repeated twice — squaring cannot create a new prime. So if p appears in a^2, it was already inside a.

2.5 The Proof Method: Contradiction

All irrationality proofs in this chapter use one recipe:

Step 1 — Assume the opposite: suppose the number IS rational, so it equals a/b with a, b integers, b ≠ 0, and a, b coprime (no common factor except 1).

Step 2 — Square / rearrange to force a prime to divide both a and b.

Step 3 — This clashes with 'a and b are coprime'. The assumption was wrong, so the number is irrational.

For numbers like $3 + 2\sqrt{5}$: assume it is rational, isolate $\sqrt{5}$, and show $\sqrt{5}$ would then be rational — contradiction, since $\sqrt{5}$ is proved irrational.

2.6 Three Mistakes That Cost Marks Every Year

✗ Mistake 1: Using HCF × LCM = product for THREE numbers. The relation holds only for two.

✗ Mistake 2: Forgetting to state 'a and b are coprime' at the start of the proof. Without it, the contradiction has nothing to clash with — examiners deduct here.

✗ Mistake 3: Ends-with-zero questions. A number ends in 0 only if BOTH 2 and 5 are in its prime factorisation. $6^n = 2^n \times 3^n$ has no 5, so it can never end in 0 — by the uniqueness part of FTA, no 5 can ever appear.

PART 3 ● SECTION-WISE QUESTION BANK

Section A1 — Multiple Choice Questions (1 mark each)

Answer key with reasons follows the questions.

Q1. The HCF of 26 and 91 is: *[Frequently Asked]*

(a) 2 (b) 7 (c) 13 (d) 26

Q2. The prime factorisation of 3825 is: *[NCERT-based]*

(a) $3 \times 5^2 \times 17$ (b) $3^2 \times 5^2 \times 17$ (c) $3^2 \times 5 \times 17$ (d) $3^2 \times 5^2 \times 7$

Q3. If HCF(a, b) = 12 and a × b = 1800, then LCM(a, b) is: *[Frequently Asked]*

(a) 120 (b) 150 (c) 180 (d) 3600

Q4. The exponent of 2 in the prime factorisation of 144 is:

(a) 2 (b) 3 (c) 4 (d) 5

Q5. If two positive integers are $a = x^3y^2$ and $b = xy^3$ (x, y prime), then HCF(a, b) is: *[Competency]*

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

Q6. For the same $a = x^3y^2$ and $b = xy^3$ as above, LCM(a, b) is: *[Competency]*

(a) xy^2 (b) x^3y^3 (c) x^4y^5 (d) x^3y^2

Q7. The LCM of the smallest prime number and the smallest odd composite number is: *[Tricky]*

(a) 9 (b) 12 (c) 18 (d) 20

Q8. If two numbers are coprime, then their LCM equals:

(a) their HCF (b) 1 (c) their product (d) their sum

Q9. 7 × 11 × 13 + 13 is: *[PYQ-style]*

(a) a prime number (b) a composite number (c) an irrational number (d) neither prime nor composite

Q10. The digit at the units place of 6^n (n a natural number) is always: *[Competency]*

(a) 0 (b) 2 (c) 4 (d) 6

Q11. If LCM(x, 18) = 36 and HCF(x, 18) = 2, then x is: *[PYQ-style]*

(a) 2 (b) 3 (c) 4 (d) 5

Q12. The product of a non-zero rational number and an irrational number is: *[Frequently Asked]*

(a) always rational (b) always irrational (c) rational or irrational (d) always an integer

Q13. Given HCF(306, 657) = 9, the LCM(306, 657) is: *[NCERT Exercise]*

(a) 22338 (b) 20342 (c) 22334 (d) 23328

Q14. Three bells toll at intervals of 9, 12 and 15 minutes. If they toll together now, after how much time will they next toll together? *[Case-style MCQ]*

(a) 60 minutes (b) 120 minutes (c) 180 minutes (d) 360 minutes

Q15. If p is a prime number and p divides a^2, then p divides:

(a) a (b) a + 1 (c) 2a only (d) cannot be said

Answer Key — Section A1 (with one-line reasons)

Q	Answer	Reason

1	(c) 13	26 = 2 × 13, 91 = 7 × 13; common prime is 13.
2	(b) $3^2 \times 5^2 \times 17$	3825 ÷ 3 = 1275, ÷ 3 = 425, ÷ 5 = 85, ÷ 5 = 17.
3	(b) 150	LCM = (a × b) ÷ HCF = 1800 ÷ 12 = 150.
4	(c) 4	$144 = 2^4 \times 3^2$.
5	(b) xy^2	Smallest powers of common primes: x^1 and y^2.
6	(b) x^3y^3	Greatest powers: x^3 and y^3.
7	(c) 18	Smallest prime = 2, smallest odd composite = 9; LCM(2, 9) = 18.
8	(c) their product	HCF = 1, so LCM = (a × b) ÷ 1 = a × b.
9	(b) a composite number	13(7 × 11 + 1) = 13 × 78 — has a factor other than 1 and itself.
10	(d) 6	$6^n = 2^n \times 3^n$ contains no 5, so it never ends in 0; powers of 6 always end in 6.
11	(c) 4	x × 18 = LCM × HCF = 36 × 2 = 72, so x = 4.
12	(b) always irrational	If r × s were rational (r rational ≠ 0), then s = (r × s)/r would be rational — contradiction.
13	(a) 22338	LCM = (306 × 657) ÷ 9 = 201042 ÷ 9 = 22338.
14	(c) 180 minutes	$9 = 3^2$, $12 = 2^2 \times 3$, $15 = 3 \times 5$; LCM = $2^2 \times 3^2 \times 5 = 180$.
15	(a) a	Standard theorem: squaring creates no new primes, so p was already a factor of a.

Section A2 — Assertion-Reason Questions (1 mark each)

Instructions (as printed in the board paper)

(a) Both Assertion (A) and Reason (R) are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is NOT the correct explanation of A.

(c) A is true, but R is false.

(d) A is false, but R is true.

Q16. Assertion (A): $\sqrt{5}$ is an irrational number. Reason (R): The sum of a rational number and an irrational number is always irrational. *[AR]*

Q17. Assertion (A): For any three positive integers, HCF × LCM = product of the three numbers. Reason (R): For any two positive integers a and b, HCF(a, b) × LCM(a, b) = a × b. *[AR • Trap]*

Q18. Assertion (A): 6^n can never end with the digit 0 for any natural number n. Reason (R): The prime factorisation of 6^n contains only the primes 2 and 3, and a number ending in 0 must have 5 as a prime factor. *[AR]*

Q19. Assertion (A): 2 is the only even prime number. Reason (R): Every even number is a composite number. *[AR]*

Q20. Assertion (A): Two numbers cannot have 500 as their HCF if their LCM is 1200. Reason (R): The HCF of two numbers always divides their LCM exactly. *[AR • Competency]*

Answer Key — Section A2

Q16 → (b). Both statements are true, but the irrationality of $\sqrt{5}$ is proved by contradiction using prime factorisation, not by the sum property. R does not explain A. *(1 mark)*

Q17 → (d). A is false — the relation fails for three numbers (e.g., HCF(6, 72, 120) × LCM(6, 72, 120) = 6 × 360 ≠ 6 × 72 × 120). R is true. *(1 mark)*

Q18 → (a). Both true, and R is exactly why A holds: by the uniqueness of prime factorisation, no 5 can appear in $6^n = 2^n \times 3^n$. *(1 mark)*

Q19 → (c). A is true. R is false — 2 itself is even but prime, so not every even number is composite. *(1 mark)*

Q20 → (a). Both true, and R explains A: since HCF must divide LCM, and 500 does not divide 1200, HCF = 500 is impossible. *(1 mark)*

Section B — Short Answer Type I (2 marks each)

Q21. Find the HCF and LCM of 510 and 92, and verify that HCF × LCM = product of the two numbers. *[NCERT Exercise]*

Q22. Check whether 6^n can end with the digit 0 for any natural number n. Justify using the Fundamental Theorem of Arithmetic. *[NCERT Exercise • Frequently Asked]*

Q23. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers. *[NCERT Exercise]*

Q24. Given that HCF(306, 657) = 9, find LCM(306, 657) without factorising the numbers again. *[NCERT Exercise]*

Q25. Two positive integers are given as $p = ab^2$ and $q = a^3b$, where a and b are prime numbers. Find HCF(p, q) and LCM(p, q). *[Competency • PYQ-style]*

Q26. Find the HCF of 96 and 404 by the prime factorisation method. Hence find their LCM. *[NCERT Example]*

Answer Key — Section B (step-wise, marking-scheme style)

Answer 21:

$510 = 2 \times 3 \times 5 \times 17$; $92 = 2^2 \times 23$ *(½ mark)*

HCF = 2 (only common prime, smallest power) *(½ mark)*

LCM = $2^2 \times 3 \times 5 \times 17 \times 23 = 23460$ *(½ mark)*

Verification: HCF × LCM = $2 \times 23460 = 46920$ and $510 \times 92 = 46920$. Verified. *(½ mark)*

Answer 22:

A number ends with 0 only if it is divisible by 10, i.e., both 2 and 5 must appear in its prime factorisation. *(1 mark)*

$6^n = (2 \times 3)^n = 2^n \times 3^n$. By the uniqueness of the Fundamental Theorem of Arithmetic, 2 and 3 are the ONLY primes in 6^n; 5 never appears. Hence 6^n can never end with the digit 0. *(1 mark)*

Answer 23:

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times 78 = 13 \times 2 \times 3 \times 13$. Since it has factors other than 1 and itself, it is composite. *(1 mark)*

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times 1009$. Again more than two factors, hence composite. *(1 mark)*

Answer 24:

For two numbers, LCM = (product of the numbers) ÷ HCF. *(1 mark)*

LCM(306, 657) = $(306 \times 657) \div 9 = 201042 \div 9 = 22338$. *(1 mark)*

Answer 25:

$p = a^1b^2$ and $q = a^3b^1$. *(½ mark)*

HCF = smallest powers of common primes = $a^1 \times b^1 = ab$. *(¾ mark)*

LCM = greatest powers of all primes = $a^3 \times b^2 = a^3b^2$. *(¾ mark)*

Answer 26:

$96 = 2^5 \times 3$; $404 = 2^2 \times 101$. *(1 mark)*

HCF = $2^2 = 4$. *(½ mark)*

LCM = $(96 \times 404) \div 4 = 38784 \div 4 = 9696$. *(½ mark)*

Section C — Short Answer Type II (3 marks each)

Q27. Prove that $\sqrt{5}$ is an irrational number. *[NCERT Exercise • Board Favourite]*

Q28. Prove that $3 + 2\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational. *[NCERT Exercise • Frequently Asked]*

Q29. Prove that $1/\sqrt{2}$ is irrational. *[NCERT Exercise]*

Q30. Find the HCF and LCM of 6, 72 and 120 using the prime factorisation method. Also check whether HCF × LCM equals the product of the three numbers. *[NCERT Example]*

Q31. There is a circular path around a sports field. Sonia takes 18 minutes to complete one round while Ravi takes 12 minutes. If both start together at the same point, in the same direction and at the same time, after how many minutes will they meet again at the starting point? *[NCERT Exercise • Case-style]*

Q32. Find the largest number which divides 245 and 1029, leaving remainder 5 in each case. *[HOTS-lite • PYQ-style]*

Answer Key — Section C (step-wise)

Answer 27:

Assume, to the contrary, that $\sqrt{5}$ is rational. Then $\sqrt{5} = a/b$, where a and b are integers, $b \neq 0$, and a, b are coprime. *(1 mark)*

Then $b\sqrt{5} = a$. Squaring: $5b^2 = a^2$. So 5 divides a^2, and since 5 is prime, 5 divides a. Write $a = 5c$. *(1 mark)*

Substituting: $5b^2 = 25c^2$, so $b^2 = 5c^2$. Thus 5 divides b^2 and hence 5 divides b. Now 5 divides both a and b — this contradicts that a, b are coprime. Hence our assumption is wrong: $\sqrt{5}$ is irrational. *(1 mark)*

Answer 28:

Assume $3 + 2\sqrt{5}$ is rational. Then $3 + 2\sqrt{5} = a/b$ for integers a, b ($b \neq 0$). *(1 mark)*

Rearranging: $2\sqrt{5} = a/b - 3 = (a - 3b)/b$, so $\sqrt{5} = (a - 3b)/(2b)$. *(1 mark)*

The right side is a ratio of integers, hence rational — but $\sqrt{5}$ is irrational. This contradiction means our assumption is false, so $3 + 2\sqrt{5}$ is irrational. *(1 mark)*

Answer 29:

Assume $1/\sqrt{2}$ is rational, say $1/\sqrt{2} = a/b$, with a, b integers, $b \neq 0$, $a \neq 0$. *(1 mark)*

Then $\sqrt{2} = b/a$, which is a ratio of integers and hence rational. *(1 mark)*

This contradicts the fact that $\sqrt{2}$ is irrational. So $1/\sqrt{2}$ is irrational. *(1 mark)*

Answer 30:

$6 = 2 \times 3$; $72 = 2^3 \times 3^2$; $120 = 2^3 \times 3 \times 5$. *(1 mark)*

$HCF = 2^1 \times 3^1 = 6$ (smallest powers of common primes 2 and 3). *(½ mark)*

$LCM = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$ (greatest powers of all primes). *(½ mark)*

Check: HCF × LCM = 6 × 360 = 2160, while 6 × 72 × 120 = 51840. They are not equal — the relation HCF × LCM = product holds only for two numbers, not three. *(1 mark)*

Answer 31:

They meet again at the start after a time that is a common multiple of both round-times; the first such time is the LCM. *(1 mark)*

$18 = 2 \times 3^2$; $12 = 2^2 \times 3$. $LCM = 2^2 \times 3^2 = 36$. *(1 mark)*

So Sonia and Ravi meet again at the starting point after 36 minutes (Sonia completes 2 rounds, Ravi 3 rounds). *(1 mark)*

Answer 32:

The required number divides 245 − 5 = 240 and 1029 − 5 = 1024 exactly. So we need HCF(240, 1024). *(1 mark)*

$240 = 2^4 \times 3 \times 5$; $1024 = 2^{10}$. *(1 mark)*

$HCF = 2^4 = 16$. The largest such number is 16. (Check: 245 = 16 × 15 + 5 and 1029 = 16 × 64 + 5.) *(1 mark)*

Section D — Long Answer Type (5 marks each)

Q33. Prove that $\sqrt{2}$ is an irrational number. Hence, show that $5 + 3\sqrt{2}$ is also irrational. *[Board Favourite]*

Q34. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 - 2\sqrt{3}$ is also irrational. *[Board Favourite]*

Q35. (a) State the Fundamental Theorem of Arithmetic. (b) Using it, find the HCF and LCM of 336 and 54, and verify that HCF × LCM = product of the two numbers. (c) Give one example to show that this verification cannot be extended to three numbers. *[Mixed Concept]*

Q36. (a) A school library has 96 English, 240 Hindi and 336 Mathematics books. The librarian wants to stack them so that all books are stored subject-wise and every stack has the same height (same number of books). Find the maximum number of books in each stack and the total number of stacks. (b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds. If they all change together at 7:00:00 am, at what time will they next change together?
[Application • Case-style]

Answer Key — Section D (step-wise)

Answer 33:

Part 1 — Assume $\sqrt{2}$ is rational: $\sqrt{2} = a/b$, a and b integers, $b \neq 0$, a and b coprime. *(1 mark)*

Then $2b^2 = a^2$.

$\Longrightarrow$ 2 divides a^2.

$\Longrightarrow$ 2 divides a (2 is prime). Write $a = 2c$. *(1 mark)*

Substituting: $2b^2 = 4c^2$.

$\Longrightarrow$ $b^2 = 2c^2$.

$\Longrightarrow$ 2 divides b. So 2 divides both a and b, contradicting coprimality. Hence $\sqrt{2}$ is irrational. *(1 mark)*

Part 2 — Assume $5 + 3\sqrt{2}$ is rational, say p/q. Then $3\sqrt{2} = p/q - 5 = (p - 5q)/q$, so $\sqrt{2} = (p - 5q)/(3q)$. *(1 mark)*

The right side is rational, but $\sqrt{2}$ is irrational — contradiction. Hence $5 + 3\sqrt{2}$ is irrational. *(1 mark)*

Answer 34:

Part 1 — Assume $\sqrt{3} = a/b$ with a, b coprime integers, $b \neq 0$. *(1 mark)*

$3b^2 = a^2$.

$\Longrightarrow$ 3 divides a^2.

$\Longrightarrow$ 3 divides a. Put $a = 3c$. *(1 mark)*

Then $3b^2 = 9c^2$.

$\Longrightarrow$ $b^2 = 3c^2$.

$\Longrightarrow$ 3 divides b. Both a and b divisible by 3 contradicts coprimality. So $\sqrt{3}$ is irrational. *(1 mark)*

Part 2 — Assume $7 - 2\sqrt{3}$ is rational = p/q. Then $2\sqrt{3} = 7 - p/q = (7q - p)/q$, so $\sqrt{3} = (7q - p)/(2q)$, a rational number. *(1 mark)*

This contradicts Part 1. Hence $7 - 2\sqrt{3}$ is irrational. *(1 mark)*

Answer 35:

(a) Every composite number can be expressed as a product of primes, and this factorisation is unique except for the order of the prime factors. *(1 mark)*

(b) $336 = 2^4 \times 3 \times 7$; $54 = 2 \times 3^3$. HCF = $2 \times 3 = 6$; LCM = $2^4 \times 3^3 \times 7 = 3024$. *(2 marks)*

Verification: $6 \times 3024 = 18144$ and $336 \times 54 = 18144$. Verified. *(1 mark)*

(c) Take 6, 72, 120: HCF = 6, LCM = 360, so HCF × LCM = $2160 \neq 6 \times 72 \times 120 = 51840$. The relation fails for three numbers. *(1 mark)*

Answer 36:

(a) Equal stacks subject-wise $\Longrightarrow$ books per stack must divide 96, 240 and 336; the maximum is HCF(96, 240, 336). *(½ mark)*

$96 = 2^5 \times 3$; $240 = 2^4 \times 3 \times 5$; $336 = 2^4 \times 3 \times 7$. HCF $= 2^4 \times 3 = 48$ books per stack. *(1½ marks)*

Stacks: $96 \div 48 = 2$, $240 \div 48 = 5$, $336 \div 48 = 7$. Total stacks $= 2 + 5 + 7 = 14$. *(1 mark)*

(b) They change together after LCM(48, 72, 108) seconds. $48 = 2^4 \times 3$; $72 = 2^3 \times 3^2$; $108 = 2^2 \times 3^3$. LCM $= 2^4 \times 3^3 = 432$ seconds. *(1½ marks)*

432 s = 7 minutes 12 seconds. Next simultaneous change: 7:07:12 am. *(½ mark)*

Section E — Case-Study Based Questions (4 marks each: 1 + 1 + 2)

Case Study 1 — Republic Day March-Past

For the Republic Day parade practice, three students are asked to march starting from the same point. Their step lengths are 40 cm, 42 cm and 45 cm respectively. The PT teacher wants each of them to cover the same distance in complete steps, using the minimum possible distance.

Q37(i). Write the prime factorisations of 42 and 45.

Q37(ii). What concept — HCF or LCM — is needed to find the minimum common distance? Give a one-line reason.

Q37(iii). Find the minimum distance each student should walk, in metres. OR If a fourth student with a step of 36 cm joins, what is the new minimum distance?

Case Study 2 — Tiling the School Hall

The school is renovating a rectangular hall of length 18 m and breadth 12 m. The contractor suggests covering the floor with identical square tiles of the largest possible size, without cutting any tile.

Q38(i). Convert the dimensions into centimetres and state which concept (HCF/LCM) gives the largest square tile side.

Q38(ii). Find the side of the largest square tile.

Q38(iii). How many such tiles will cover the hall completely? OR If tiles of side 300 cm are used instead, how many tiles are needed?

Case Study 3 — School Bells

Three bells in a school ring at intervals of 9 minutes, 12 minutes and 15 minutes. All three bells ring together at 8:00 am when school opens.

Q39(i). Write 9, 12 and 15 as products of prime factors.

Q39(ii). After how many minutes will the three bells ring together again?

Q39(iii). At what time will they ring together next? How many times will they ring together after 8:00 am, up to and including 8:00 pm? OR If the third bell's interval changes to 10 minutes, after how long will all three ring together?

Answer Key — Section E (step-wise)

Answer 37:

(i) $42 = 2 \times 3 \times 7$; $45 = 3^2 \times 5$. *(1 mark)*

(ii) LCM — the required distance must be a common multiple of all three step lengths, and we want the smallest such distance. *(1 mark)*

(iii) $40 = 2^3 \times 5$. LCM(40, 42, 45) $= 2^3 \times 3^2 \times 5 \times 7 = 2520$ cm = 25.2 m. *(2 marks)*

OR: $36 = 2^2 \times 3^2$. New LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$ cm (36's primes are already covered), so the distance stays 25.2 m. *(2 marks)*

Answer 38:

(i) 18 m = 1800 cm, 12 m = 1200 cm. The largest tile side must divide both dimensions exactly $\Longrightarrow$ HCF. *(1 mark)*

(ii) $1800 = 2^3 \times 3^2 \times 5^2$; $1200 = 2^4 \times 3 \times 5^2$. HCF $= 2^3 \times 3 \times 5^2 = 600$ cm, i.e., a 6 m square tile. *(1 mark)*

(iii) Tiles along length $= 1800 \div 600 = 3$; along breadth $= 1200 \div 600 = 2$. Total tiles $= 3 \times 2 = 6$. *(2 marks)*

OR: With 300 cm tiles: $(1800 \div 300) \times (1200 \div 300) = 6 \times 4 = 24$ tiles. *(2 marks)*

Answer 39:

(i) $9 = 3^2$; $12 = 2^2 \times 3$; $15 = 3 \times 5$. *(1 mark)*

(ii) LCM $= 2^2 \times 3^2 \times 5 = 180$ minutes. *(1 mark)*

(iii) 180 minutes = 3 hours, so next together at 11:00 am. From 8:00 am to 8:00 pm is 720 minutes; together at 180, 360, 540 and 720 minutes $\Longrightarrow$ 4 times (11:00 am, 2:00 pm, 5:00 pm, 8:00 pm). *(2 marks)*

OR: With intervals 9, 12, 10: $10 = 2 \times 5$; LCM $= 2^2 \times 3^2 \times 5 = 180$ minutes — still 3 hours. *(2 marks)*

Bonus Section — HOTS / Toppers' Zone (Above Board Level)

These stretch the same concepts further. Perfect for students targeting 95+ or preparing for competitive exams alongside boards.

Q40. Prove that $\sqrt{p}$ is irrational for every prime number p. *[HOTS • Generalisation]*

Q41. Find the smallest number which, when divided by 35, 56 and 91, leaves remainder 7 in each case. *[HOTS]*

Q42. If the HCF of 65 and 117 can be expressed in the form 65m − 117, find the value of m. *[HOTS • PYQ-style]*

Q43. Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number. *[HOTS]*

Q44. Find the greatest 6-digit number that is exactly divisible by 24, 15 and 36. *[HOTS]*

Answer Key — HOTS

Answer 40:

Assume $\sqrt{p} = a/b$ with a, b coprime integers, $b \neq 0$. Then $pb^2 = a^2$, so p divides a^2 and hence p divides a (p prime). Put a = pc.

Then $pb^2 = p^2c^2$.

$\implies b^2 = pc^2$.

$\implies$ p divides b. So p divides both a and b — contradicting coprimality. Hence $\sqrt{p}$ is irrational for every prime p.

Answer 41:

The required number is 7 more than a common multiple of 35, 56 and 91; the smallest is LCM + 7.

$35 = 5 \times 7$; $56 = 2^3 \times 7$; $91 = 7 \times 13$. LCM $= 2^3 \times 5 \times 7 \times 13 = 3640$.

Smallest number = 3640 + 7 = 3647.

Answer 42:

$65 = 5 \times 13$ and $117 = 3^2 \times 13$, so HCF(65, 117) = 13.

$65m - 117 = 13$.

$\implies 65m = 130$.

$\implies m = 2$.

Answer 43:

Assume $\sqrt{2} + \sqrt{3} = r$, a rational number. Squaring: $2 + 2\sqrt{6} + 3 = r^2$, so $\sqrt{6} = (r^2 - 5)/2$.

The right side is rational, so $\sqrt{6}$ would be rational. But $6 = 2 \times 3$ is not a perfect square and $\sqrt{6}$ is irrational (provable by the standard contradiction method with the prime 2: $6b^2 = a^2$ forces $2 \mid a$ and then $2 \mid b$).

Contradiction $\implies \sqrt{2} + \sqrt{3}$ is irrational.

Answer 44:

Any number divisible by all three must be divisible by LCM(24, 15, 36). $24 = 2^3 \times 3$; $15 = 3 \times 5$; $36 = 2^2 \times 3^2$. LCM $= 2^3 \times 3^2 \times 5 = 360$.

Greatest 6-digit number = 999999. Dividing: $999999 = 360 \times 2777 + 279$.

Required number = 999999 − 279 = 999720.

How to Use This Chapter for Maximum Marks

1. Read the Exam Radar first — know exactly what the board asks before you practise.

2. Master ONE irrationality proof (Q27 or Q33) until you can write it in under 4 minutes. Every other proof reuses the same skeleton.

3. Practise all three case studies with a timer — 7 minutes each. Section E is where prepared students pull ahead.

4. Revisit the 'Three Mistakes' list one day before the exam.